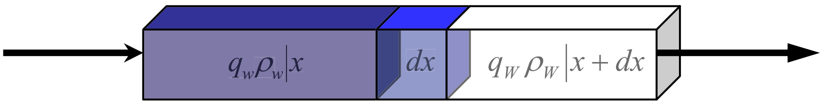
Classic Waterflooding Predicitive Models

Initial water injection to water breakthrough: Buckley-Leverett

Buckley and Leverett (1942) developed a mathematical approach to describe two-phase, immiscible displacement in a linear system.

In a differential element of porous media, the frontal advance theory maintains that mass is conserved: Volume of fluid entering – Volume of fluid leaving = Change in fluid volume



The mathematical development of the Buckley-Leverett frontal advance theory can be found in any petroleum engineering textbook. We will concentrate on the practical application. Although developed for waterflooding applications, the model is applicable to other fluids, including polymer, gels, surfactants, etc.

The BL theory includes several important assumptions:

- Single layer homogeneous reservoir
- Capillary pressure effects are negligible
- Linear flow
- No free gas saturation in the reservoir at any time
- Incompressible fluids

At mobility ratios <1, the BL piston-like displacement theory is correct, but may not be valid at mobility ratios greater than about 10 due to the effects of viscous fingering.

The following data is taken from Craft and Hawkins (1959).

Oil formation volume factor (Bo)	1.25 bbl/STB
Water formation volume factor (Bw)	1.02 bbl/STB
Formation thickness (h)	20 ft
Cross sectional area (A)	26,400 ft^2
Porosity (Φ)	25%
Injection rate (i_w)	900 bbl/day
Distance between producer and injector (L)	660ft (20 ac)
Oil viscosity (μ_o)	2.0 cp
Water viscosity (μ_w)	1.0 cp
Dip angle (α)	0º
Connate water saturation (S_{wc})	20%
Initial Water Saturation (S_{wi})	20%
Residual oil saturation (S_{or})	20%
Relative Permeability vs. Water Saturation:	

Sw	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
K_{ro}/k_{rw}	30.23	17.00	9.56	5.38	3.02	1.70	0.96	0.54	0.30	0.17	0.10

Table 1

The linear flow and single homogenous layer assumptions implicit in the Buckley-Leverett theory translate to areal and vertical sweep efficiencies of 100%. However, inspection of Figure 1 illustrates that in practice areal coverage and vertical heterogeneity must be considered in waterflooding calculations. In practice, areal and vertical sweep efficiencies typically range from 70% to 100% and 40% to 80%, respectively (World Oil 1966). Figure 2 shows a linear flow approximation to a five-spot pattern. Areal and vertical sweep efficiency will be discussed in more detail later.

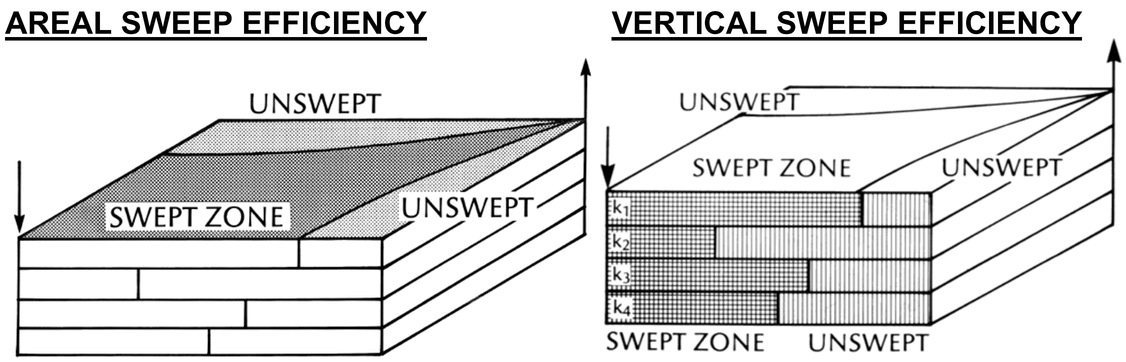


Figure 1

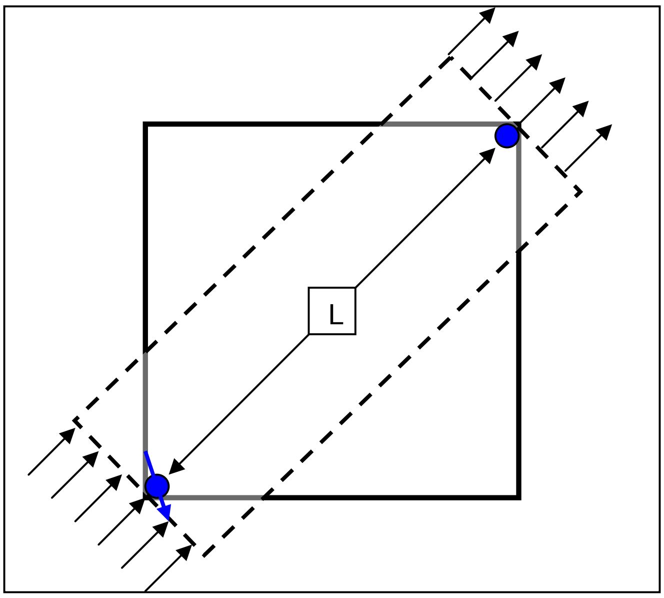


Figure 2

The following calculations assume that water injection begins with initial oil production. However, the principles are equally valid if the OOIP is reduced by a primary production factor.

<u>Problem</u>:

a) Calculate and plot the water saturation profile after 60, 120 and 240 days.
b) Calculate time to breakthrough
c) Cumulative water injected at breakthrough
d) Water volume (pore volume) injected at breakthrough

<u>Solutions</u>:

Step 1: Plot the relative permeability ratio k_{ro}/k_{rw} vs. water saturation on a semi-log scale. The relative permeability vs. Sw curve can be described mathematically as:

$$\frac{k_{ro}}{k_{rw}} = ae^{bS_w} \qquad \text{Eq. 1}$$

Where,

k_{ro} = relative permeability to oil

k_{rw} = relative permeability to water

S_W = water saturation at the production wells

From the linear segment of the graph of Kro/Krw vs. Sw (Figure 3), Excel calculates an exponential trend line with a slope of -11.47 and an intercept of 529.39 (the values of "a" and "b" in Eq. 8).

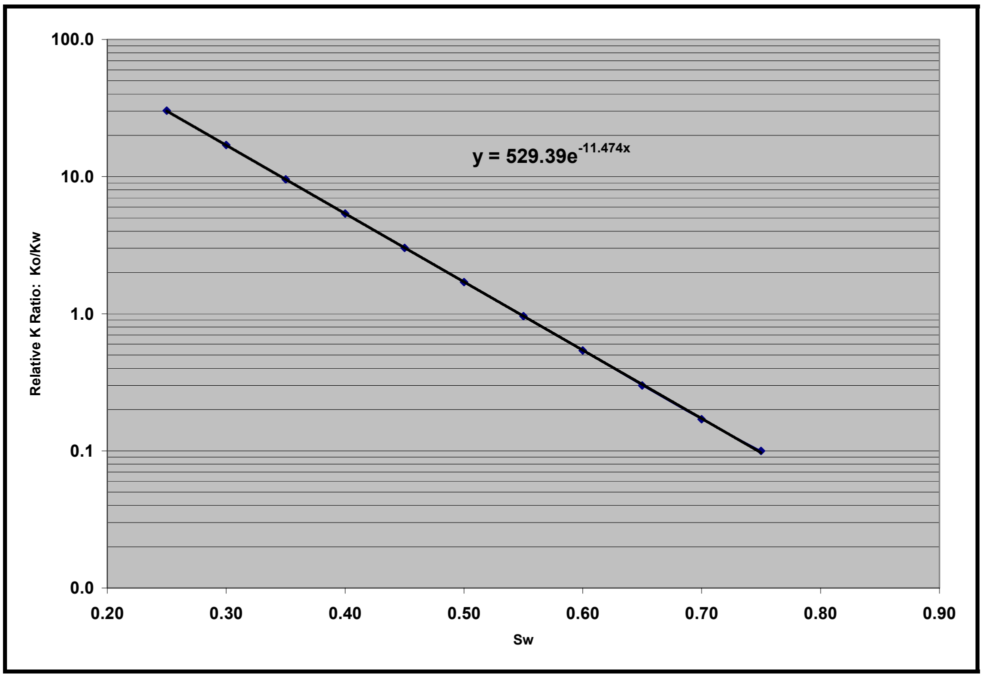


Figure 3: Relative Permeability Ratio vs. Water Saturation

Step 2:
Assume several Sw values and calculate the fractional flow curve at its derivatives using the following equation:

$$fw = \frac{1}{1 + \left(\frac{\mu_w}{\mu_o}\right) ae^{bS_w}} \qquad \text{Eq. 2}$$

Where fw is the producing water cut

Differentiating Eq. 2 with respect to Sw give the slope of the fractional flow curve:

$$\left(\frac{df_w}{dS_w}\right)_{Sw} = -\frac{\left(\frac{\mu_W}{\mu_O}\right) abe^{bSw}}{\left[1 + \left(\frac{\mu_W}{\mu_O}\right) ae^{bSw}\right]^2} \qquad \text{Eq. 3}$$

Step 3: Plot fw and (dfw/Sw) vs. Sw (Figure 4)

.

Sw	Kro/Krw	fw	dFw/dSw	
0.25	30.06	0.062	0.671	
0.30	16.94	0.106	1.084	
0.35	9.54	0.173	1.644	
0.40	5.38	0.271	2.267	
0.45	3.03	0.398	2.748	
0.50	1.71	0.540	2.851	
0.55	0.96	0.675	2.516	
0.596	0.57	0.779	1.975	← water breakthrough
0.60	0.54	0.787	1.925	
0.65	0.30	0.868	1.318	
0.70	0.17	0.921	0.837	
0.75	0.10	0.954	0.506	

Sw, kro/krw: Table 1
fw: Eq. 2
dfw/dSw: Eq. 3

Table 2

.

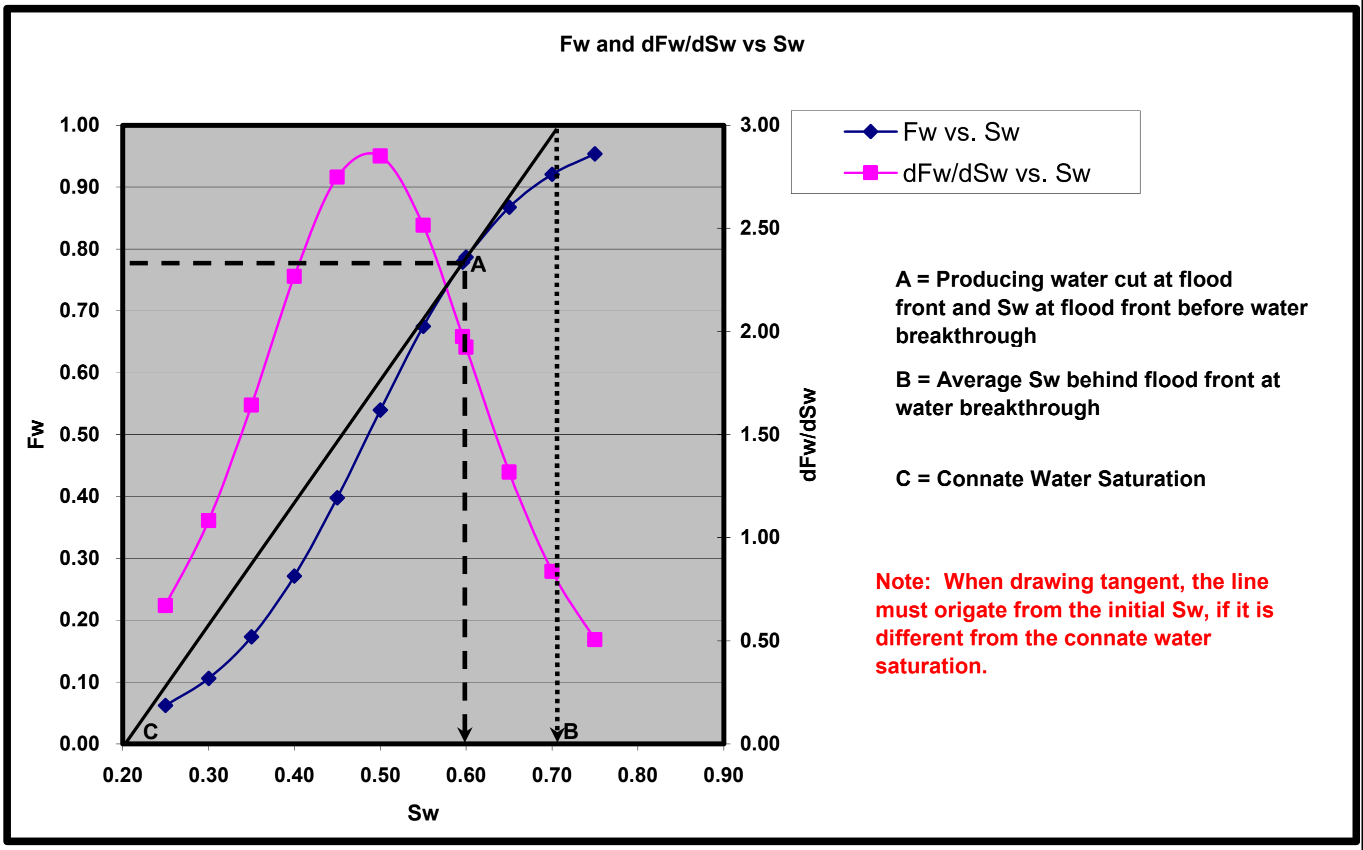


Fig. 4: Fractional Flow Curve

Figure 4 indicates that the leading edge of the flood front has a water saturation of 59.6%, which means that the water saturation behind the flood front has a minimum water saturation of 59.6%.

Step 4:
Assuming water saturations from 60% to 75% (75% = 1-Sor), calculate the oil bank saturation profile using the following equation:

$$(x)S_w = \left(\frac{5.615 i_W t}{\phi A}\right)\left(\frac{df_W}{dS_W}\right)_{S_W} \qquad \text{Eq. 4}$$

Using the values given in the data set above, this reduces to:

$$(x)S_w = \left(\frac{(5.615)(900)t}{(0.25)(26400)}\right)\left(\frac{df_W}{dS_W}\right)_{S_W} = (0.77t)\left(\frac{df_W}{dS_W}\right)_{S_W} \qquad \text{Eq. 5}$$

The exercise on the following page illustrates the physical locations of the flood front in the reservoir for t = 60, 120 and 240 days after initial water injection.

Using Eq. 5 and the data from Table 2 above, we can calculate the distance (feet) from the injection well to the producing well at Sw from 60% to 75%.

dFw/dSw	t = 60	t = 120	t = 240	Sw
1.925	88	177	354	0.60
1.318	61	121	242	0.65
0.837	38	77	154	0.70
0.506	23	46	93	0.75

Table 3

Now we can visualize the flood front at 60, 120 and 240 days:

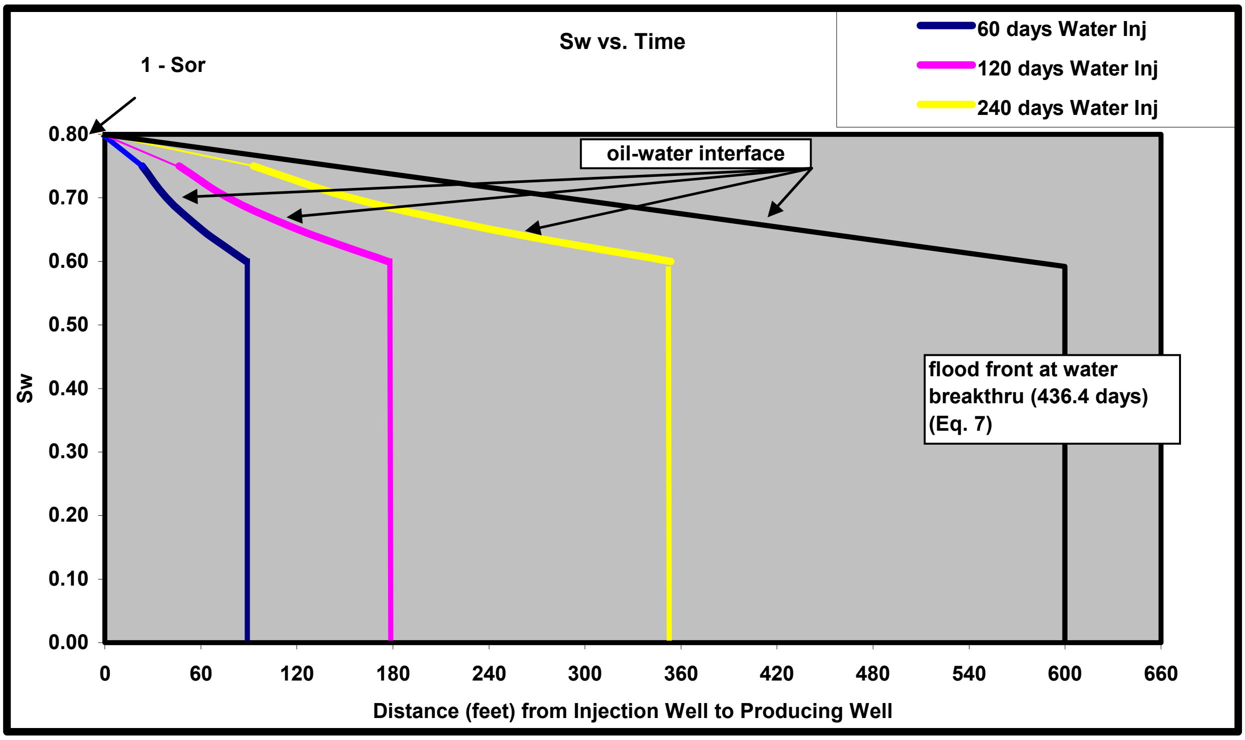


Figure 5

The flood front will eventually reach the producing well at which time water breakthrough will occur. Note that the value of the water saturation in the water-invaded portion of the reservoir at the time the water breaks through to the producing well will be about 70% (Point B in Figure 4).

Time to water breakthrough

Pore volume (PV) is given by:

$$PV = \frac{\phi A L}{5.615} \qquad \text{Eq. 6}$$

The time to water breakthrough, t_{BT}, can be estimated using Eq. 4, setting $(x)S_W = L = 600\,ft$ Recall that L is the distance from the injector to producer. Combining Equations 4 and 6 and solving for time (t):

$$t_{BT} = \left[\frac{(PV)}{i_W}\right]\left(\frac{1}{\frac{df_W}{dS_W}}\right)_{S_{Wf}}$$ Eq. 7

Using the data given,

$$PV = \frac{(0.25)(20ac)(20ft)}{\frac{5.615ft^3}{bbl}} \times \frac{43560ft^2}{ac} = 775{,}779bbl$$

$$OOIP = Np = \frac{PV(1-S_{wi})}{B_o} = \frac{775{,}779 \times (1-.20)}{1.25} = 496{,}499bbls$$ Eq 8

$$t_{BT} = \left[\frac{(775{,}779)}{900}\right]\left(\frac{1}{1.975}\right) = 436.4days$$

Estimated Cumulative water injected at breakthrough (W_{iBT})

$$W_{iBT} = i_W t_{BT}$$ Eq. 9

$$W_{iBT} = (900)(436.4) = 392{,}760bbl$$

Estimated total PV of water injected at breakthrough (Q_{iBT})

$$Q_{iBT} = \frac{1}{\left(\frac{df_W}{dS_W}\right)_{S_{wf}}}$$ Eq. 10

$$Q_{iBT} = \frac{1}{1.975} = 0.506PV$$

Note: Since Buckley-Leverett theory assumes that mass is conserved, the volume of oil displaced at water breakthrough is equal to the volume of water injected: 403,200 reservoir barrels, or 0.52 PV.

Surface WOR at breakthrough (WOR_s)

$$WOR_s = \frac{B_o f_w}{B_w(1 - f_w)} \qquad \text{Eq. 11}$$

$$WOR_s = \frac{(1.25)(0.779)}{(1.02)(1 - 0.779)} = 4.32$$

Summary of Reservoir Performance to the point of water breakthrough:

Assume $E_A = E_V = 100\%$ and gas saturation = 0.

t, days	W_{inj} = 900t	$N_P = \frac{W_{inj}}{B_o}$	$Q_o = \frac{i_w}{B_o}$	WOR_s	Q_w ($Q_o WOR_s$)	W_p
0	0	0	0	0	0	0
30	27000	21600	720	0	0	0
60	54000	43200	720	0	0	0
90	81000	64800	720	0	0	0
120	108000	86400	720	0	0	0
150	135000	108000	720	0	0	0
180	162000	129600	720	0	0	0
210	189000	151200	720	0	0	0
240	216000	172800	720	0	0	0
270	243000	194400	720	0	0	0
300	270000	216000	720	0	0	0
330	297000	237600	720	0	0	0
360	324000	259200	720	0	0	0
390	351000	280800	720	0	0	0
420	378000	302400	720	0	0	0
436	392400	313920	720	0	0	0
436.4**	392760	314208	159	4.32	687	0

Table 4

** Water breakthrough

After water breakthrough: Welge

After water breakthrough, Welge (1952) demonstrated that the following parameters can be determined from the fractional flow curve:

- Surface water cut, fw2
- Average Water saturation in the reservoir, $\overline{S_{w2}}$
- Cumulative water injected, Qi

Welge's principles will be illustrated as part of the following example.

Example Oil Recovery Calculations (After Primary Oil Production)

Using the reservoir data presented in Table 1, construct a set of performance curves to predict the waterflood performance up to a surface WOR of 45 (economic limit).
Assume $E_A = E_V$ = 100% and gas saturation = 0.

Recall the following parameters at breakthrough calculated above:

$S_{wf} = S_{wBT}$= 0.60 Figure 2
$f_{wf} = f_{wBT}$ = 0.78 Figure 2
$(dfw/dSw)_{BT}$ = 1.975 Table 2
Q_{ibt} = 1/1.973 = 0.52 PV Equation 9
$\overline{S_{wBT}}$ = 0.70 Figure 4 (point B)
$(Np)_{BT}$ = 314208 STB Table 4
W_{iBT} = 392,760 BBL Table 4
t_{BT} = 436.4 days Table 4
WOR_s = 4.32 Equation 11

Now, we can construct a table for reservoir performance after water breakthrough:

Sw2	fw2	dfw/dSw	Sw2 ave	Ed	Np	Qi	Winj	t	Wp	WORs	Qo	Qw
0.596	**0.787**	**1.975**	**0.70**	**0.63**	**314208**	**0.51**	**392799**	**436**	**0**	**4.3**	**159**	**687**
0.60	**0.787**	**1.925**	**0.71**	**0.64**	**316995**	**0.52**	**403080**	**448**	**6703**	**4.5**	**153**	**694**
0.61	**0.805**	**1.798**	**0.72**	**0.65**	**321612**	**0.56**	**431433**	**479**	**28841**	**5.1**	**140**	**711**
0.62	**0.823**	**1.673**	**0.73**	**0.66**	**326402**	**0.60**	**463690**	**515**	**54596**	**5.7**	**128**	**726**
0.63	**0.839**	**1.551**	**0.73**	**0.67**	**331345**	**0.64**	**500277**	**556**	**84407**	**6.4**	**116**	**740**
0.64	**0.854**	**1.432**	**0.74**	**0.68**	**336425**	**0.70**	**541674**	**602**	**118767**	**7.2**	**105**	**753**
0.65	**0.868**	**1.318**	**0.75**	**0.69**	**341628**	**0.76**	**588429**	**654**	**158230**	**8.0**	**95**	**765**
0.66	**0.880**	**1.210**	**0.76**	**0.70**	**346939**	**0.83**	**641156**	**712**	**203415**	**9.0**	**86**	**777**
0.67	**0.892**	**1.107**	**0.77**	**0.71**	**352347**	**0.90**	**700552**	**778**	**255018**	**10.1**	**78**	**787**
0.68	**0.902**	**1.011**	**0.78**	**0.72**	**357842**	**0.99**	**767399**	**853**	**313820**	**11.3**	**70**	**796**
0.69	**0.912**	**0.921**	**0.79**	**0.73**	**363413**	**1.09**	**842577**	**936**	**380697**	**12.7**	**63**	**805**
0.70	**0.921**	**0.837**	**0.79**	**0.74**	**369054**	**1.20**	**927078**	**1030**	**456628**	**14.2**	**57**	**812**
0.71	**0.929**	**0.759**	**0.80**	**0.75**	**374756**	**1.32**	**1022016**	**1136**	**542717**	**16.0**	**51**	**820**
0.72	**0.936**	**0.687**	**0.81**	**0.77**	**380512**	**1.45**	**1128641**	**1254**	**640197**	**17.9**	**46**	**826**
0.73	**0.943**	**0.621**	**0.82**	**0.78**	**386318**	**1.61**	**1248359**	**1387**	**750453**	**20.1**	**41**	**832**
0.74	**0.948**	**0.561**	**0.83**	**0.79**	**392167**	**1.78**	**1382748**	**1536**	**875039**	**22.5**	**37**	**837**
0.75	**0.954**	**0.506**	**0.84**	**0.80**	**398054**	**1.98**	**1533579**	**1704**	**1015698**	**25.3**	**33**	**842**
0.76	**0.959**	**0.456**	**0.85**	**0.81**	**403976**	**2.20**	**1702840**	**1892**	**1174382**	**28.4**	**30**	**846**
0.77	**0.963**	**0.410**	**0.86**	**0.83**	**409929**	**2.44**	**1892762**	**2103**	**1353285**	**31.8**	**27**	**850**
0.78	**0.967**	**0.368**	**0.87**	**0.84**	**415909**	**2.71**	**2105847**	**2340**	**1554864**	**35.7**	**24**	**853**
0.79	**0.970**	**0.331**	**0.88**	**0.85**	**421914**	**3.02**	**2344905**	**2605**	**1781875**	**40.0**	**21**	**856**
0.80	**0.973**	**0.297**	**0.89**	**0.86**	**427941**	**3.37**	**2613085**	**2903**	**2037412**	**44.9**	**19**	**859**

Table 5

The first line of data represents the point of water breakthrough.

Where,

<u>Col 1</u>: Sw2 = water saturation values at the production wells after water breakthrough
These are assumed values in order to complete the rest of Table 5.

<u>Col 2</u>: fw2 = producing water cut after water breakthrough (Eq. 2)

<u>Col 3</u>: dfw/dSw = slope of fractional flow curve after water breakthrough (Eq. 3)

<u>Col 4</u>: Sw2ave = average water saturation in the reservoir after water breakthrough

$$Sw2ave = Sw2 + \frac{1 - fw2}{\left(\frac{dfw}{dSw}\right)} \quad \text{Eq. 12}$$

<u>Col 5</u>: Ed = displacement efficiency

$$Ed = \frac{Sw2ave - Swi}{1 - Swi} \quad \text{Eq. 13}$$

Recall that Swi is the initial reservoir water saturation given in Table 1

<u>Col 6</u>: Np = Cumulative Oil production, bbls.

$$Np = OOIP \times E_D \times E_A \times E_V \quad \text{Eq. 14}$$

As mentioned above, we are assuming $E_A = E_V = 100\%$, so Np reduces to:

$$Np = OOIP \times E_D$$

<u>Col 7</u>: Qi = PV of water injected

$$Qi = \frac{1}{\left(\frac{dfw2}{dSw}\right)} \quad \text{Eq. 15}$$

<u>Col 8</u>: Winj = Cumulative water injected, bbls

$$Winj = PV \times Qi \quad \text{Eq. 16}$$

PV is calculated above (Eq. 6): 775,779 bbl

<u>Col 9</u>: t = time (days) to inject Winj

$$t = \frac{Winj}{i_w} \quad \text{Eq. 17}$$

i_w is the water injection rate given in Table 1: 900 bbl/day

Col 10: Wp = Cumulative water production

Recall that under the material balance equation, the cumulative water injected is equal to the cumulative production of oil + water. Another key assumption, stated earlier, is that no free gas saturation exists in the reservoir.

$$Wp = \frac{Winj - (Sw2ave - Swi) \times (PV) \times E_A \times E_V}{B_W} \qquad \text{Eq. 18}$$

B_w is given in Table 1

Col 11: WORs = Surface water-oil ratio

$$WORs = \frac{B_O}{B_W \times \left(\frac{1}{fw2} - 1 \right)} \qquad \text{Eq. 19}$$

B_O is given in Table 1

Col 12: Qo = Oil flow rate, surface bbls/day

$$Qo = \frac{i_W}{B_O + \left(B_W \times WOR_S \right)} \qquad \text{Eq. 20}$$

Col 13: Qw = Water flow rate, surface bbls/day

$$Q_W = Q_O \times WOR_S \qquad \text{Eq. 21}$$

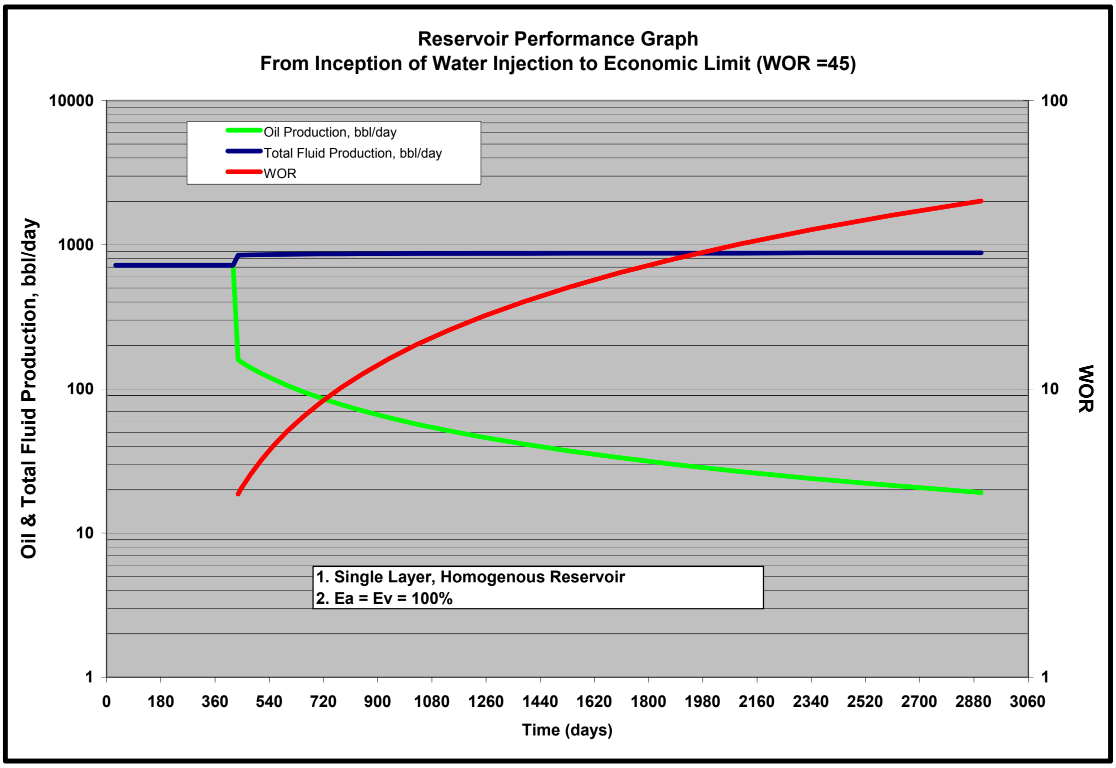
Figure 4 is a graphical representation of Tables 4 and 5. Note that the total oil recovery the economic limit is 427,945 surface barrels. The OOIP was previously computed as 496,499 surface barrels (Eq 8). Therefore, the oil recovery at a WOR of 45 is a remarkable 86% of OOIP!

427,945/496,499 = 86%

However, recall that this model includes two key assumptions that have a profound effect on oil recovery:

- Single, homogeneous layer reservoir (i.e, vertical sweep efficiency = 100%)
- Areal sweep efficiency = 100%

In the following paragraphs, we will examine the effect of each of these assumptions on reservoir performance.



Reservoir Performance Graph
From Inception of Water Injection to Economic Limit (WOR =45)
Oil Production, bbl/day
Total Fluid Production, bbl/day
WOR
Oil & Total Fluid Production, bbl/day
WOR
1. Single Layer, Homogenous Reservoir
2. Ea = Ev = 100%
Time (days)
10000
1000
100
10
1
0
180
360
540
720
900
1080
1260
1440
1620
1800
1980
2160
2340
2520
2700
2880
3060

Figure 6

Areal Sweep Efficiency

Up to this point, we have assumed that Areal sweep efficiency (E_A), is 100%. E_A is the horizontal portion of the reservoir that is contacted by water and is primarily a function of the following variables:

- Mobility Ratio
- Reservoir heterogeneity (anisotrophy)
- Cumulative volume of water injected
- Waterflood pattern configuration

A detailed discussion of the mathematics and theory of areal sweep efficiency is beyond the scope of this discussion. However, the following general observations will help develop the example that follows.

1. Water mobility (krw/µw) increases after water breakthrough due to the increase in the average reservoir water saturation and its continuity from the injection wells to the offset producing wells;
2. Lower mobility ratios will increase areal sweep efficiency while higher mobility ratios will decrease it.
3. Studies have shown that continued water injection can, over time, significantly increase areal sweep efficiency, particularly in reservoirs with an adverse mobility ratio.
4. In a tilted reservoir, areal efficiency is improved when the injection well is located downdip (displacing oil updip).
5. Examples of reservoir heterogeneities that are always present to some degree include:
    a. Permeability anisotrophy (directional permeability);
    b. Fractures;
    c. Flow barriers;
    d. Uneven permeability/porosity distribution.

As mentioned earlier, extensive waterflood experience in the United States indicates that areal sweep efficiency after breakthrough varies from 70%--100%. E_A typically increases from zero at the time of initial water injection until water breakthrough. After water breakthrough, E_A continues to increase, although at a slower rate.

Example Calculation, Areal Sweep Efficiency

We will use the same data from Table 1. In addition, assume the following relative permeability data, which corresponds to the kro/krw ratios given in Table 1:

Sw	kro	Krw
0.25	0.500	0.017
0.30	0.370	0.022
0.35	0.280	0.029
0.40	0.210	0.039
0.45	0.150	0.050
0.50	0.100	0.059
0.55	0.070	0.073
0.60	0.048	0.089
0.65	0.032	0.107
0.70	0.032	0.135
0.75	0.018	0.180

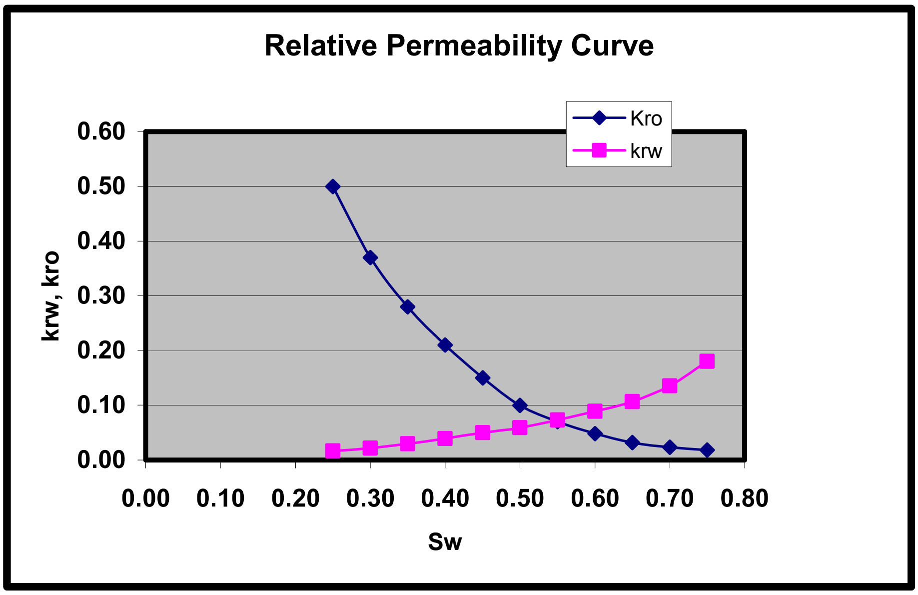


Figure 7

Selected Data from Table 1 and previous calculations:
Gas saturation (Sg) =0; Vertical Sweep Efficiency (Ev) = 100%. Recall that we are still assuming a single layer, homogeneous reservoir.
B_o = 1.25 bbl/STB
B_w = 1.02 bbl/STB
μ_o = 2 cp
μ_w=1 cp
S_{wi}= 0.20
i_w = 900 bbl/day
PV = 775,779 bbl (Eq. 6)

Additional data needed:

Mobility ratio (M) = $\frac{Krw_{BT} * \mu_o}{Kro_{BT} \cdot \mu_w} = \frac{0.089 * 2}{0.048 * 1} = 3.70$ Eq. 22

K_{rw} and K_{ro} values are the relative permeability values given in the table above at S_w = 0.596. Recall that this is the water saturation at the flood front (see Figure 4).

The methodology presented in Table 6 is described by Ahmed, 2001.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Winj	t	Wi/Wibt	E_A	Qi/Qibt	Qi	dfw/dSw	Sw2	fw2	Sw2 ave	E_D	Np	Wp	WORs	Qo	Qw
213349	237	1.00	0.54	1.00	0.51	1.98	0.60	0.79	0.70	0.63	169894	0	1.10	379	418
240349	267	1.13	0.58	1.10	0.55	1.80	0.61	0.81	0.72	0.65	185118	8776	1.33	345	460
267349	297	1.25	0.61	1.28	0.65	1.54	0.63	0.84	0.73	0.67	200702	16149	1.64	308	505
321349	357	1.51	0.66	1.45	0.73	1.36	0.65	0.87	0.75	0.68	222724	42102	2.16	260	563
375349	417	1.76	0.70	1.69	0.86	1.17	0.67	0.89	0.76	0.70	243899	69094	2.73	223	609
456349	507	2.14	0.75	1.91	0.97	1.03	0.68	0.90	0.77	0.72	268252	118662	3.39	191	648
537349	597	2.52	0.80	2.20	1.11	0.90	0.69	0.91	0.79	0.73	290781	170464	4.06	167	678
618349	687	2.90	0.84	2.46	1.25	0.80	0.70	0.92	0.80	0.75	310517	225689	4.74	148	701
699349	777	3.28	0.87	2.72	1.37	0.73	0.71	0.93	0.81	0.76	328068	283593	5.44	132	720
807349	897	3.78	0.91	3.02	1.53	0.65	0.72	0.94	0.82	0.77	348565	364356	6.32	117	739
915349	1017	4.29	0.94	3.31	1.68	0.60	0.73	0.94	0.83	0.78	366766	447933	7.24	104	755
1023349	1137	4.80	0.97	3.59	1.82	0.55	0.74	0.95	0.83	0.79	383200	533676	8.19	94	767
1131349	1257	5.30	1.00	3.87	1.96	0.51	0.75	0.95	0.84	0.80	397255	622334	9.20	85	779

Table 6

The first line represents the point of water breakthrough.

A detailed explanation of each column follows. The results of the sample calculations have, in some cases, been forced to agree to Table 6 values due to immaterial rounding differences from the spreadsheet calculations.

Col 1: First, E_A at breakthrough must be estimated. Willhite (1986) presents the following correlation:

$$E_{A_{BT}} = 0.54602036 + \frac{0.03170817}{M} + \frac{0.30222997}{e^{M}} - 0.00509693M \qquad \text{Eq. 23}$$

$$E_{A_{BT}} = 0.54602036 + \frac{0.03170817}{3.70} + \frac{0.30222997}{e^{3.70}} - 0.00509693 * 3.70 = 0.543$$

Next, calculate PV of water injected at breakthrough (Qi_{BT}). From Eq. 15,

$Qi = \frac{1}{\left(\frac{dfw2}{dSw}\right)}$; at breakthrough $Qi_{BT} = \frac{1}{1.975} = 0.51$ (Col 6, line 1)

Now, the volume of water injected at breakthrough is

$PV * Qi_{BT} * E_{A_{BT}} = 775{,}779bbl * 0.506 * 0.543 = 213{,}349bbl$ (Col 1, line 1) Eq. 24

Subsequent values of W_{inj} are arbitrary increments.

Col 2:

$t = W_{inj} / i_w$ Eq. 25

Example, line 1: $t = 213{,}349 / 900 = 237days$

Col 3: Self explanatory. Example, line 5:

$$\frac{W_i}{W_{ibt}} = \frac{375{,}349bbl}{213{,}349bbl} = 1.76 \qquad \text{Eq. 26}$$

Col 4: E_A at breakthrough is given in Eq. 22. E_A after breakthrough can be calculated from the following equation:

$$E_A = E_{A_{BT}} + 0.2749 * \ln\left(\frac{W_i}{W_{i_{BT}}}\right)$$ Eq. 27

Example, line 5:

$$E_A = 0.543 + 0.2749 \ln\left(\frac{375{,}349}{213{,}349}\right) = 0.70$$

Col 5: Q_i/Q_{iBT} values for values of E_{ABT} are taken from Appendix E of SPE Monograph Volume 3 (Craig, 1971)

Col 6: Q_{iBT} is calculated above (0.51).

After breakthrough, $Q_{iBT} = \left(\frac{Q_i}{Q_{iBT}}\right) * Q_{iBT}$ Eq. 28

Example, line 5:

$$Q_i = 1.691 * 0.51 = 0.86$$

$\left(\frac{Q_i}{Q_{iBT}}\right)$ is from Table E.9, page 120 of Craig (1971)

Col 7: $\left(\frac{dfw}{dSw}\right)$ is the slope of the fractional flow curve.

$\left(\frac{dfw}{dSw}\right)_{BT}$ is calculated from Eq. 3.

The value at breakthrough (1.975, line 1) is from Table 2.

After breakthrough, Q_i is simply the reciprocal of Eq. 15:

Example, line 5: $\left(\frac{dfw}{dSw}\right) = \frac{1}{Q_i} = \frac{1}{0.856} = 1.17$

Col 8: Sw2 is the water saturation at the producing well. At breakthrough ($Sw2_{BT}$ =0.596) is determined from Figure 4. After breakthrough, Sw2 is estimated by taking the nearest value of Sw2 from Table 5 that corresponds to each value of $\left(\frac{dfw}{dSw}\right)$ in Table 6.

Col 9: fw2 is the producing well water cut for each value of Sw2. All values of fw are from Table 5, as determined from Eq. 2.

Col 10: $Sw2_{ave}$ is the average water saturation in the swept portion of the reservoir. At breakthrough, ($Sw2_{ave}$=0.70) is from Figure 4. After breakthrough, the equation is

$$Sw2_{ave} = Sw2 + \frac{1 - fw2}{\left(\frac{dfw}{dSw}\right)}$$ Eq. 29

Example calculation, line 5:

$$Sw2_{ave} = 0.67 + \frac{1 - 0.892}{1.168} = 0.76$$

Col 11: E_D is the displacement efficiency for each value of $Sw2_{ave}$ and is given by Eq. 13. For example, for line 5 we calculate E_D as follows:

$$Ed = \frac{Sw2ave - Swi}{1 - Swi} = \frac{0.76 - .020}{1 - 0.20} = 0.70$$

Col 12: From Eq. 14, $Np = OOIP \times E_D \times E_A \times E_V$
Recall that up to this point we are assuming that E_V = 1.0

From Eq. 8, OOIP = 496,499 STB
Example, line 5: $Np = 496{,}499 \times 0.70 \times 0.70 \times 1.0 = 243{,}899 STB$

Col 13: Cumulative water production, Wp, is computed from the following equation:

$$Wp = \frac{W_i - ((Sw2_{ave} - Sw_i) * PV * E_A)}{B_w}$$ Eq. 30

Example, line 5:

$$Wp = \frac{375{,}349 - ((0.76 - 0.20) * 775{,}779 * 0.70)}{1.03} = 69{,}094 bbl$$

Col 14: After water breakthrough, there are two sources of oil production: Oil that is being displaced behind the flood front in the swept layers plus oil from newly swept layers. Craig et al. (1955) developed the following equation to express the incremental oil from the newly swept zones:

$$(\Delta Np)_{NEW} = E\lambda$$

Where,

$$E = \frac{Sw2_{BT} - Sw_i}{E_{A_{BT}} * (Sw2_{ave_{BT}} - Sw_i)}$$ and Eq. 31

$$\lambda = 0.2749 * \left(\frac{Wi_{BT}}{Wi}\right)$$ Eq. 32

Craig et. al (1955) then proposed that the surface water/oil ratio WOR_s is given by:

$$WOR_S = \left[\frac{fw2 * [1 - (\Delta Np_{NEW})]}{1 - [fw2 * (1 - (\Delta Np_{NEW}))]}\right] * \left(\frac{B_O}{B_W}\right)$$ Eq. 33

Observe that when E_A reaches 100%, $(\Delta Np)_{NEW}$ becomes 0. The parameter λ decreases with increasing water injection.
Example, Col 14, line 5:

$$E = \frac{0.596 - 0.20}{0.543 * (0.70 - 0.20)} = 1.45$$

Note that E is a constant.

$$\lambda = 0.2749 * \left(\frac{213{,}349}{375{,}349} \right) = 0.156$$

$$(\Delta Np)_{NEW} = 1.45 * 0.156 = 0.226;$$

Finally, line 5 of Col. 14 is:

$$WOR_S = \frac{0.892 * (1 - 0.226)}{1 - [0.892 * (1 - 0.226)]} * \left(\frac{1.25}{1.02} \right) = 2.73$$

<u>Col 15</u>: Oil rate, Qo

$$Q_o = \frac{i_w}{B_O + (B_w * WOR_S)} \qquad \text{Eq. 34}$$

Example, line 5:

$$Q_o = \frac{900}{1.25 + (1.02 * 2.73)} = 223 STB/d$$

<u>Col 16</u>: Water Rate, Qw

$$Q_W = Q_o * WOR_S \qquad \text{Eq. 35}$$

Example, line 5:

$$Q_W = 223 * 2.73 = 609 STB/d$$

We could continue developing Table 6 after E_A reaches 1.0. However, the primary objective of the above exercise is to get a sense of how E_A progresses to unity with the volume of water injected.

Recall that we began with a single layer, homogeneous reservoir with the explicit assumptions that E_V = E_A = 100% and S_g =0. Next we developed the calculations for areal sweep efficiency (E_A). In the following case, we will study the effects of vertical sweep efficiency (E_V) and gas saturation (S_g).

Stratified Reservoirs

All oil and gas reservoirs are stratified to some degree. Various methodologies have been proposed to forecast waterflood performance in layered reservoirs. Stiles (1949) proposed an approach that has been widely accepted. The Stiles method includes the following simplifying assumptions:

- The layers are of constant thickness and are continuous between the injection well and offset producing wells;
- Linear system with no crossflow or segregation of fluids in the layers;
- Piston-like displacement with no oil produced behind the flood front
- Constant porosity and fluid saturations
- In all layers, the same relative permeability to oil ahead of the flood front and relative permeability to water behind the flood front.
- Except for absolute permeability, the reservoir rock and fluid characteristics are the same in all layers
- The position of the flood front in a layer is directly proportional the absolute permeability of the layer

Tiorco's experience demonstrates that the Stiles method will generate a reasonably accurate history match and production forecast in a multi-layer reservoir up to mobility ratios of about 10.

We are now ready to develop a comprehensive example that incorporates reservoir heterogeneity and well as an adverse mobility ratio. The following example, using the reservoir rock and fluid properties of the El Tordillo field described in SPE 113334, illustrates the mechanics of the Stiles method (Smith, 1966).

Assume the following reservoir characteristics and conditions at the start of the waterflood:

OOIP, STB	2,655,714
Primary Recovery, % OOIP	10%
Residual oil saturation, Sor	20%
Total net pay, ft	50
Areal Sweep Efficiency, E_A	1.0
Mobility Ratio, M	9.24
Distance between injectors, ft. (50 ac. Spacing)	1,476 ft
Ave Bo (RB/STB)	1.08
Ave steady state injection rate, bpd	1500
Gas saturation (S_g), %PV at start of waterflood	1.5%

Table 7

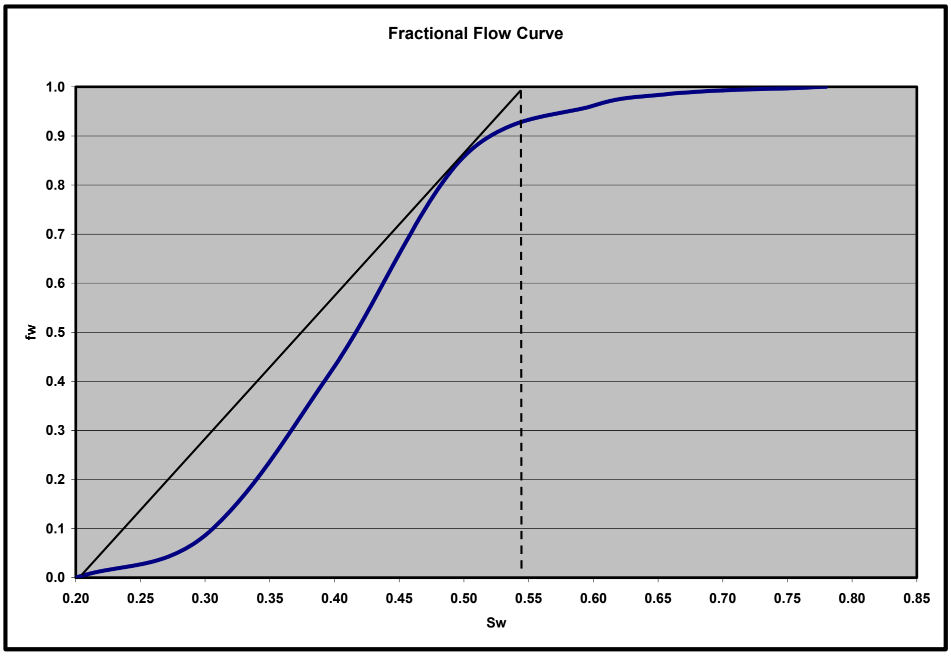


Figure 8

Using the above data, the following table is prepared:

(1) Σhj ft	(2) kj md	(3) ΣkjΔhj md-ft	(4) Σhj x kj md-ft	(5) R	(6) Np STB	(7) ΔNp STB	(8) fw	(9) qo	(10) t (days)	(11) Wi	(12) f'w
1	3541	3541	3541	0.141	261194	261194	0.275	1006.9	308	461464	0.624
2	2858	6399	5716	0.169	314729	53535	0.605	548.2	405	607949	0.776
3	2240	8639	6720	0.205	381045	66316	0.762	330.7	606	908760	0.842
4	1833	10472	7332	0.237	440886	59841	0.831	234.8	861	1291114	0.879
5	1491	11963	7455	0.273	507902	67016	0.870	180.0	1233	1849698	0.902
6	1273	13236	7638	0.303	563045	55143	0.895	145.3	1613	2419047	0.919
7	1085	14321	7595	0.335	621951	58906	0.913	120.7	2101	3151105	0.931
8	955	15276	7640	0.361	671187	49235	0.926	102.6	2581	3870915	0.941
9	847	16123	7623	0.387	718843	47656	0.936	88.4	3119	4679132	0.948
10	750	16873	7500	0.413	768535	49693	0.945	77.1	3764	5646236	0.955
11	673	17546	7403	0.438	813927	45392	0.951	67.8	4434	6650278	0.960
12	607	18153	7284	0.462	857958	44030	0.957	60.1	5166	7749607	0.964
13	555	18708	7215	0.482	896540	38583	0.961	53.5	5887	8830898	0.968
14	509	19217	7126	0.502	933883	37342	0.966	47.9	6668	10001491	0.971
15	464	19681	6960	0.524	973972	40089	0.969	42.9	7602	11403311	0.974
16	425	20106	6800	0.544	1012171	38199	0.972	38.6	8592	12888708	0.977
17	390	20496	6630	0.565	1049620	37449	0.975	34.8	9670	14504388	0.979
18	367	20863	6606	0.579	1075789	26169	0.977	31.4	10503	15754651	0.981
19	335	21198	6365	0.600	1114623	38834	0.980	28.3	11874	17811341	0.983
20	308	21506	6160	0.619	1150407	35784	0.982	25.6	13272	19908096	0.985
21	290	21796	6090	0.632	1175657	25250	0.983	23.2	14362	21543376	0.986
22	264	22060	5808	0.653	1214547	38889	0.985	20.9	16221	24331710	0.987
23	243	22303	5589	0.672	1248820	34273	0.986	18.9	18032	27048015	0.989
24	230	22533	5520	0.684	1271071	22251	0.988	17.1	19331	28996811	0.990
25	210	22743	5250	0.703	1307143	36072	0.989	15.5	21665	32497608	0.991
26	197	22940	5122	0.717	1332063	24921	0.990	14.0	23451	35176139	0.992
27	182	23122	4914	0.733	1362177	30114	0.991	12.6	25846	38769499	0.992
28	169	23291	4732	0.748	1389740	27563	0.992	11.3	28283	42425167	0.993
29	158	23449	4582	0.761	1414017	24277	0.993	10.2	30674	46011321	0.994
30	145	23594	4350	0.777	1444122	30106	0.993	9.1	33987	50980660	0.995
31	133	23727	4123	0.793	1473782	29659	0.994	8.1	37640	56459829	0.995
32	125	23852	4000	0.804	1494339	20557	0.995	7.2	40479	60718228	0.996
33	114	23966	3762	0.820	1523728	29389	0.995	6.4	45054	67581337	0.996
34	106	24072	3604	0.832	1546127	22399	0.996	5.7	48995	73492337	0.997
35	97	24169	3395	0.846	1572292	26166	0.996	5.0	54226	81338353	0.997
36	90	24259	3240	0.857	1593370	21077	0.997	4.4	59034	88551426	0.997
37	83	24342	3071	0.869	1614866	21497	0.997	3.8	64673	97009067	0.998
38	75	24417	2850	0.882	1640382	25515	0.998	3.3	72429	108643410	0.998
39	69	24486	2691	0.893	1660168	19786	0.998	2.8	79446	119168382	0.998
40	63	24549	2520	0.904	1680182	20014	0.998	2.4	87820	131729623	0.999
41	57	24606	2337	0.915	1700496	20314	0.999	2.0	97980	146970164	0.999
42	51	24657	2142	0.926	1721215	20720	0.999	1.6	110557	165835641	0.999
43	45	24702	1935	0.937	1742503	21287	0.999	1.3	126517	189776027	0.999
44	39	24741	1716	0.949	1764620	22117	0.999	1.1	147422	221133590	0.999
45	34	24775	1530	0.959	1783546	18926	0.999	0.8	170512	255767796	1.000
46	29	24804	1334	0.970	1802589	19042	1.000	0.6	201601	302401877	1.000
47	24	24828	1128	0.980	1821820	19231	1.000	0.4	245691	368535758	1.000
48	20	24848	960	0.988	1836692	14872	1.000	0.3	296878	445317233	1.000
49	16	24864	784	0.995	1849705	13013	1.000	0.2	373715	560572165	1.000
50	12	24876	600	1.000	1859000	9295	1.000	0.1	501849	752774009	1.000

Table 8

Note: The example calculations below are not always exact due to rounding; however the differences are immaterial.

Col 1 & 2 : Assume a reservoir with a total thickness of 50 feet that can be subdivided into 50 layers on the basis of core analysis. Assuming h = 1 ft for each layer (Col 1) and ordering absolute permeability in descending order (Col 2) facilitates the calculations and interpretation of the results. The Stiles methodology evaluates the reservoir from a statistical rather than a geological standpoint. Although no reservoir would be characterized in such a manner, most multi-layer reservoirs under waterflood can be described as a series of uniform strata of equal thickness as long as the number of layers is sufficiently large.

Col 3: Cumulative absolute permeability. Example, layer 10:
$kj\Delta hj = 16123 + (750 * (10 - 9)) = 16873$. In this case the term (10-9) is superfluous; however, it would be necessary for any uniform layer thickness other than h = 1 foot.

Col 4: Product of Col 1 X Col 2. For layer 10: $hj * kj = 10 * 750 = 7500$

Col 5: R = Fraction of recoverable oil produced as each layer floods out, equivalent to the fraction of the reservoir flooded out plus the layers still contributing oil production. Example, layer 10:

$$\frac{\sum h_{layer1-9}}{\sum hj} + \left(\frac{1}{(k_{layer10}) * \sum hj}\right) * \left(\sum kj\Delta hj - \sum kj\Delta hj_{layer1-9}\right) = \frac{9}{50} + \left(\frac{1}{750 * 50}\right) * (24876 - 16123) = 0.413$$

Col 6: Cumulative Oil Recovery (Np, STB). First, calculate recoverable oil at the start of the waterflood:

$$OOIP - (\%OOIP_{PRIMARY}) - Sor = 2655714 - (0.10 * 2655714) - (0.20 * 2655714) = 1{,}859{,}000STB$$

Now, Np can be calculated. Example, layer 10:
$Np = OIP_{STARTWF} * E_A * R_{layer10} = (1{,}859{,}000 * 1.0 \cdot 0.413) = 768{,}535STB$

Col 7: ΔNp = the oil contribution between the flooding out of each layer.
For layer 10: $\Delta Np = Np_{layer10} - Np_{layer9} = 768{,}535 - 718{,}843 = 49{,}693STB$

Col 8: fw = the water cut at the producing well at reservoir conditions.

Example, layer 10:

$$fw = \frac{M * \sum kj\Delta hj_{layer9}}{M * \sum kj\Delta hj_{layer9} + \left(\sum kj\Delta hj - \sum kj\Delta hj_{layer9}\right)} = \frac{9.24 * 16123}{(9.24 * 16123) + (24876 - 16123)} = 0.945$$

Col 9: A summary of the oil production rate as each layer is flooded out

Example, layer 10: $q_{o(surface)} = \frac{(1 - fw_{layer10})}{Bo} * q_{injection} = \frac{(1 - 0.945)}{1.08} * 1500 = 77.1STB/d$

Col 10: This column calculates the time (days) that correspond to the floodout of each layer. First, we calculate the gas saturation at the start of water injection. In a solution gas drive reservoir, gas saturation results when reservoir pressure falls below the bubble point pressure, typically during primary production.

a) When water injection is initiated, a water bank begins to form around the injection well. As the water bank expands, oil is displaced, forming an oil bank. Assuming radial flow, the oil banks formed around adjacent injection wells will eventually meet. This point of contact is called **interference**. Figure 8 is a graphical representation of interference between two adjacent injection well patterns.

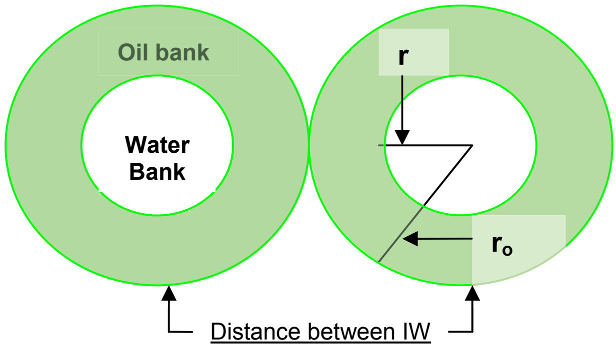


Figure 9

r_o = outer radius of oil bank
r = outer radius of water bank

Now, we can set up a table to calculate the time from initial water injection to interference. Table 7 gives the distance between injection wells as 1,476 feet. Therefore ½ of that distance, or (r_o = 738 ft), would be the point of interference assuming linear flow. Table 9 indicates that interference occurs after 38 days.

Start of water injection to interference					
(A) Winj	(B) ro	(C) r	(D) iw bwipd	(E) Δt = ΔWinj/iw(avg)	(F) days t = Σ(Δt)
1500	120	39	1500	1.00	1
10500	319	104	1500	6.00	7
21000	451	147	1500	7.00	14
31500	552	181	1500	7.00	21
42000	637	209	1500	7.00	28
52500	712	233	1500	7.00	35
56330	738	242	1500	3.00	38

Table 9

Where,
Col A: Cumulative water injected (assumed values)

Col B: Outer radius to oil bank. $r_o = \sqrt{\frac{5.615Wi}{\pi h \phi S_{gi}}}$ Eq. 36

Example, W_{inj} = 56,330 bbl

$$r_o = \sqrt{\frac{5.615 * 56{,}330}{\pi * 50 * 0.245 * 0.015}} = 738\,ft$$

Col C: Outer radius to water bank. $r = r_o \sqrt{\frac{S_{gi}}{Sw2_{ave\,BT} - Sw_i}}$ Eq. 37

Example, W_{inj} = 56,330 bbl

$$r = 738 * \sqrt{\frac{0.015}{0.54 - 0.40}} = 242\,ft$$

Col D: Average water injection rate, from Table 7
Col E: Time step, Δt

Example, W_{inj} = 56,330 bbl

$$\Delta t = \frac{\Delta W_{inj}}{i_{W_{AVE}}} = \frac{56{,}330 - 52500}{1{,}500} = 3 days$$

Col F: Cumulative days to interference, t

The total volume of water injected to fillup (W_{if}) is:
$W_{if} = PV * S_{gi} = 4{,}780{,}286 * 0.015 = 71{,}704 bbls$

Dividing by the average injection rate gives the total days to fillup.
$\frac{71{,}704}{1{,}500} = 48 days$

The final step for Col 10 is to determine the cumulative number of days.

For the first layer $t = t_{fillup} + \frac{\Delta Np}{Q_o}$; for subsequent layers, $t_n = \frac{\Delta Np}{Q_o} + t_{n-1}$

Example, layer 10
$t = 3{,}119 + \frac{49{,}693}{77.1} = 3{,}764 days$

Col 11: The volume of water injected as each layer floods out is the product of the injection rate and the days since initial water injection (t, Col 10)

$$Wi_{surface} = t * i_w$$

Example, layer 10:

$$Wi_{surface} = 3{,}764 * 1{,}500 = 5{,}646{,}236$$

Col 12: Fractional flow at surface conditions is given by:

$$f'w = \frac{\left(\frac{k_{rw} * \mu_o * B_o}{k_{ro} * \mu_w * B_w}\right) * \Sigma Col3_n}{\left(\frac{k_{rw} * \mu_o * B_o}{k_{ro} * \mu_w * B_w}\right) * \Sigma Col3_n + \left(\Sigma Col3 - \Sigma Col3_n\right)}$$

Example, layer 10:

$$f'w = \frac{\left(\frac{0.33 * 28 * 1.08}{1 * 1 * 1}\right) * 16{,}873}{\left(\frac{0.33 * 28 * 1.08}{1 * 1 * 1}\right) * 16{,}873 + \left(24{,}876 - 16{,}873\right)} = 0.955$$

Results and Comparison with SRAM

Figure 10 is included to highlight the effect of reservoir heterogeneity in the above example. Notice that about 2/3 of the injected water is sweeping only about 20% of the reservoir rock. Figures 11 and 12 compare the results of a waterflood forecast using the mathematics in this (called “SRAM II”) with Jack McCartney’s *Seconday Recovery Analysis Model* (SRAM), which was developed using the same waterflood principles and reservoir rock and fluid properties presented in the above example.

Both graphs match up well. SRAM II calculates a slightly longer fillup time and the oil response at fillup is somewhat more pronounced. After 300 months, the cumulative oil production in SRAM is 996,274 STBO vs. 998,580 STBO in SRAM II (an immaterial difference of about 0.2%). Total oil recovery after 300 months is unrealistically high in both simulations (>40% OOIP) due in part to the assumption that E_A, = 100%. However, the purpose of the exercise was to corroborate the methodology presented in this paper. A history match would have revealed that the simulation was too optimistic.

SRAM II offers a couple of advantages over our current version. First, the number of layers and the permeability of each layer can be modified to fit the reservoir under evaluation. Secondly, the gas saturation at the time of initial water injection can be modified. Both of these parameters will allow the user to better history match historical production data and tailor the simulation.

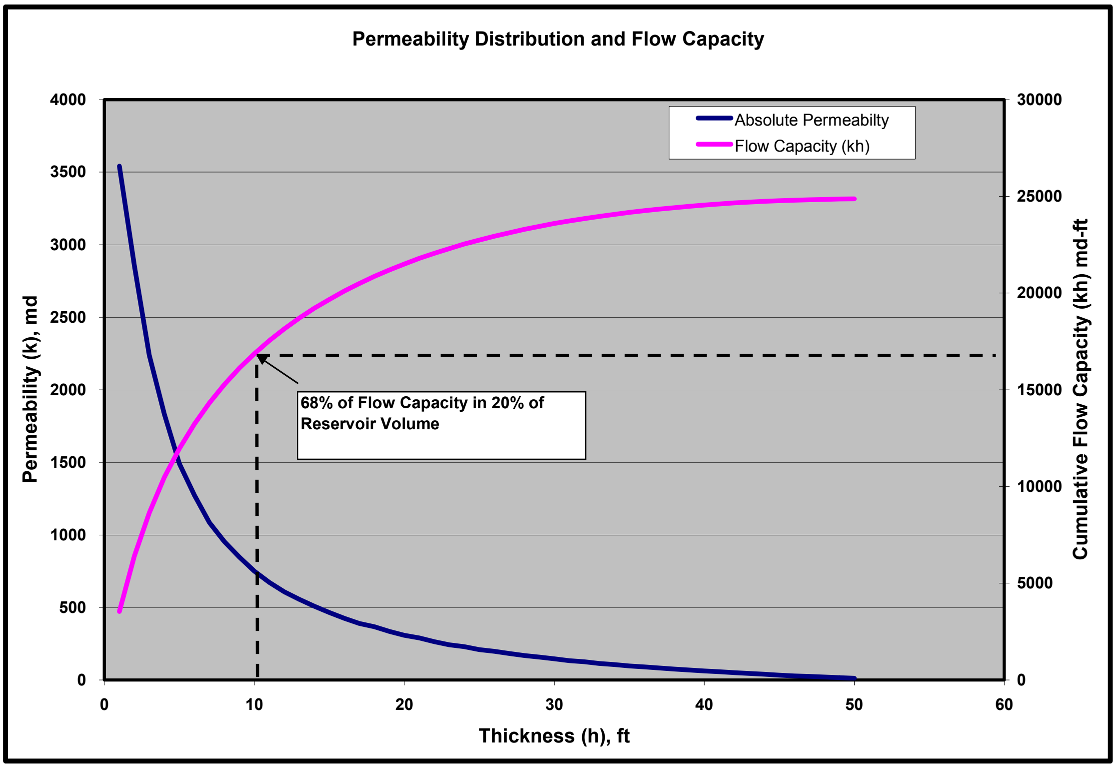


Figure 10

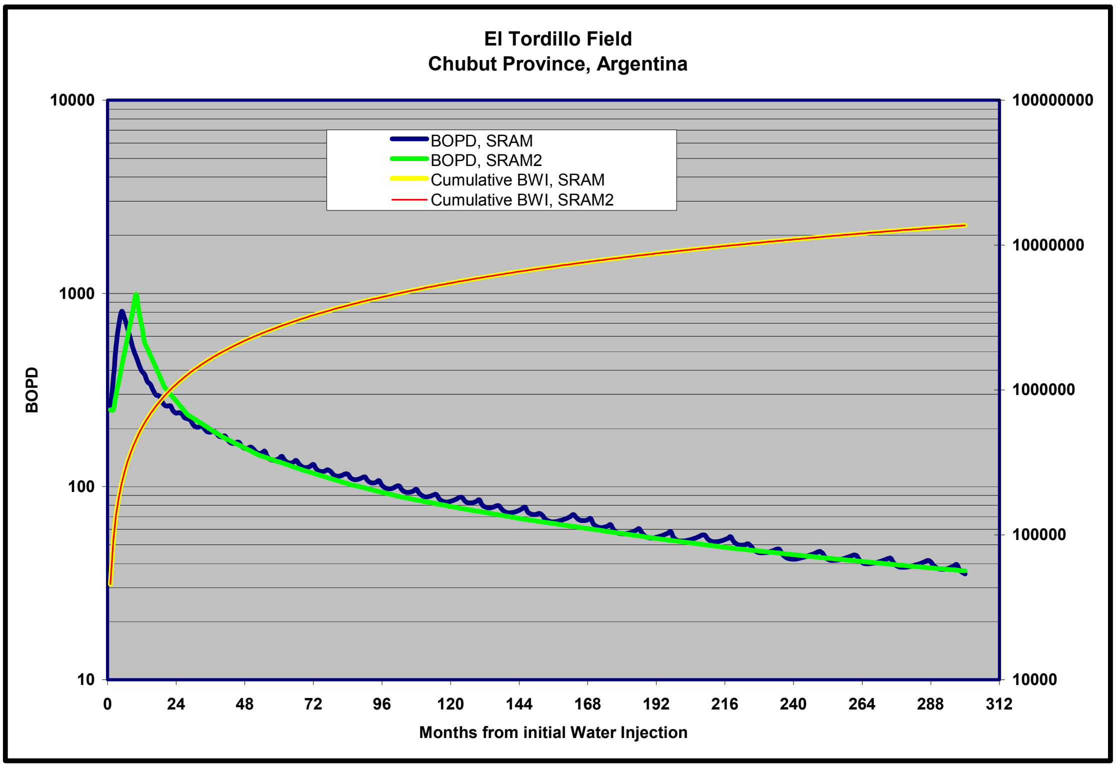


Figure 11

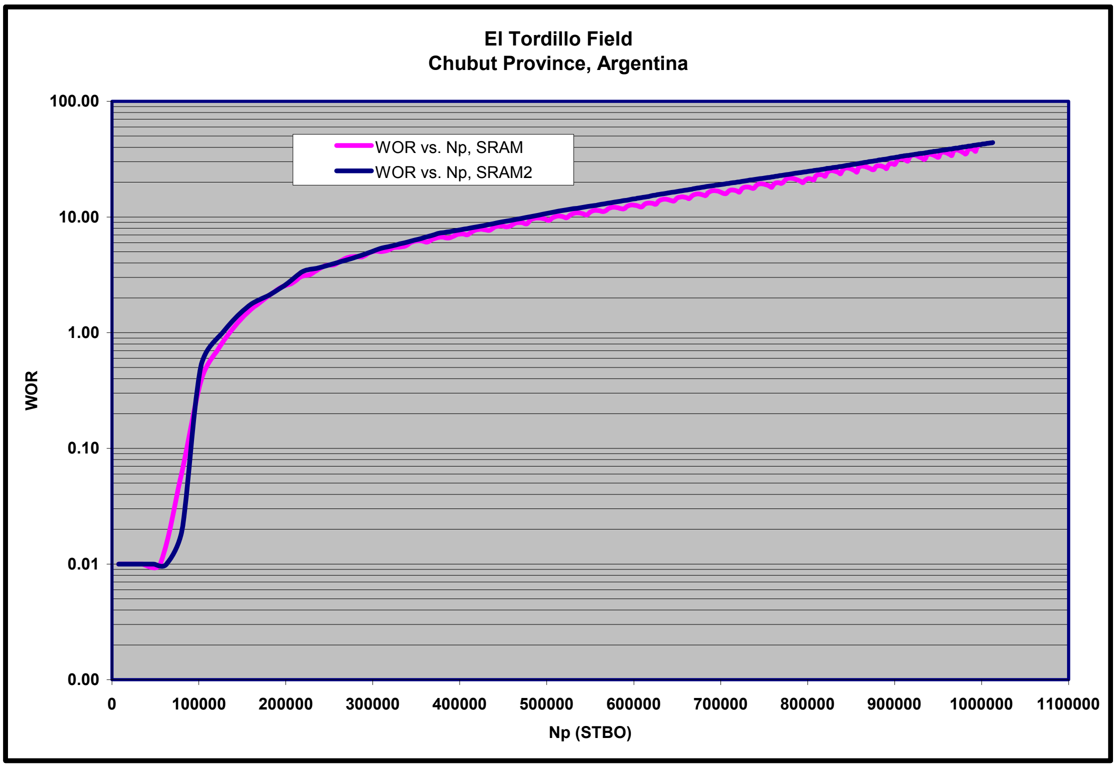


Figure 12

Conclusion

Always remember that even the most sophisticated reservoir simulators tend to give optimistic results, for a couple of reasons. First, the theories presented above include several simplifying assumptions that are necessary so that the mathematics are not overwhelming. Secondly, all the reservoir heterogeneities in a given rock volume cannot be quantified and reduced to bytes in a computer program. Always try to compare simulation results to empirical data such as historical production data trends and analogies from similar fields. Of course, a good history match is fundamental to any forecast. Question every forecast—especially your own!

You are now equipped with all the tools necessary to apply the “smell test” to any waterflood simulation or even prepare your own forecast for a multi-layer, heterogeneous reservoir. Good luck!

Bibliography

Ahmed, T., *Reservoir Engineering Handbook*, Gulf Publishing 2000.

Buckley, S.E., and Leverett, M.C*., "Mechanism of Fluid Displacement in Sands", Trans. AIME* 1942.

Craig, F., Geffen, T., and Morse, R., "Oil Recovery Performance of Pattern Gas or Water Injection Operations from Model Tests", *JPT*, Jan. 1955.

Craig, F., *The Reservoir Engineering Aspects of Waterflooding*, Society of Petroleum Engineers, 1971.

Smith, C.R., *Mechanics of Secondary Oil Recovery*, Robt. E. Krieger Publishing, 1966.

Stiles, W.E., "Use of Permeability Distribution in Waterflood Calculations,"*Trans. AIME*, 1951.

Welge, H.J., *"A Simplified Method for Computing Oil Recovery by Gas or Water Drive",Trans. AIME* 1952.

Willhite, G.P., *Waterflooding*. Society of Petroleum Engineers, 1986.

World Oil (editorial), "Practical Waterflooding Shortcuts", December 1966.